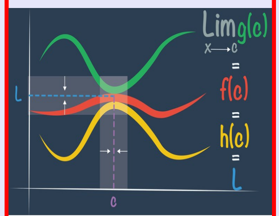
# Calculus I

# Lecture 35




Feb 19-8:47 AM

Given $f(x)=x^4-3x^3+3x^2-x$

1) Polynomial $\to$ Domain $(-\infty,\infty)$

2) Y-Int $(0,0)$

3) x-Int $(0,0)$
$(1,0)$

$x^4-3x^3+3x^2-x=0$

$x(x^3-3x^2+3x-1)=0$

$\downarrow$

$x=0$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & 3 & -1 \\ & & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & 0 \end{array}$$

$(x-1)(x^2-2x+1)=0$

$(x-1)(x-1)^2=0$

$(x-1)^3=0$

$\boxed{x=1}$ odd # of times

4) No Asymptotes

5) $f'(x)=4x^3-9x^2+6x-1$

$f'(x)=0 \quad 4x^3-9x^2+6x-1=0$

$$\begin{array}{r|rrrr} 1 & 4 & -9 & 6 & -1 \\ & & 4 & -5 & 1 \\ \hline & 4 & -5 & 1 & 0 \end{array}$$

$(x-1)(4x^2-5x+1)=0$

$(x-1)(x-1)(4x-1)=0$

$(x-1)^2(4x-1)$

C.P. $(1,0)$

$\left(\frac{1}{4}, f(\frac{1}{4})\right)$

Oct 31-7:28 AM

$f''(x) = 12x^2 - 18x + 6$     P.I.P.

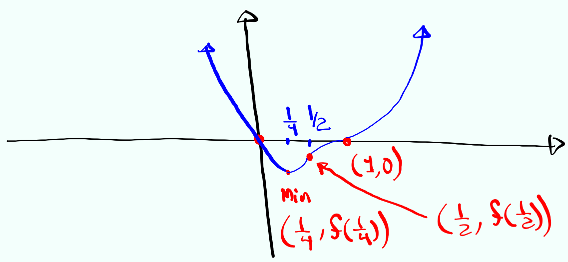
$= 6(2x^2 - 3x + 1)$     $(1, 0)$

$= 6(x-1)(2x-1)$     $\left(\frac{1}{2}, f\left(\frac{1}{2}\right)\right)$

$f'(x) = (x-1)^2(4x-1)$

| $x$ | $-\infty$ to $\frac{1}{4}$ | $\frac{1}{4}$ to $\frac{1}{2}$ | $\frac{1}{2}$ to $1$ | $1$ to $\infty$ |
|---|---|---|---|---|
| $f'(x)$ | $-$ | $+$ | $+$ | $+$ |
| $f''(x)$ | $+$ | $+$ | $-$ | $+$ |
| $f(x)$ | ↘ | ↗ | ↗ | ↗ |

Min. (at $\frac{1}{4}$)    I.P. (at $\frac{1}{2}$)    I.P. (at $1$)




Oct 31-7:37 AM

$f(x) = \frac{x^3}{x+8}$

Domain $x \neq -8 \rightarrow$ V.A. $x = -8$

Y-Int $(0,0)$   x-Int $f(x) = 0$   $\frac{x^3}{x+8} = 0$   $x^3 = 0$   $\boxed{x=0}$ odd # of times

If $\frac{x}{x+8} \rightarrow$ H.A. $y = 1$

If $\frac{x^2}{x+8} \rightarrow$ slant Asymptote

$f(x) = \frac{x^3}{x+8} \rightarrow$ Quadratic

Long Division

$$
\begin{array}{r|rrrr}
 & x^2 & -8x & +64 & \\
\hline
x+8 & x^3 & +0x^2 & +0x & +0 \\
 & -(x^3 & +8x^2) & & \\
\hline
 & & -8x^2 & +0x & +0 \\
 & & -(-8x^2 & -64x) & \\
\hline
 & & & 64x & +0 \\
 & & & -(64x & +512) \\
\hline
 & & & & -512
\end{array}
$$

$f(x) = x^2 - 8x + 64 + \frac{-512}{x+8}$

$f'(x) = 2x - 8 + \frac{512}{(x+8)^2}$

$f''(x) = 2 - \frac{1024}{(x+8)^3}$ ← Make the Correction.

C.P. $f'(x) = 0$ or $f'(x)$ is undefined

P.I.P. $f''(x)$ or $f''(x)$ is undefined

Find all C.P., and P.I.P.

Do Sign chart, then do the graph

(Finish it over weekend)

Oct 31-7:49 AM

$x>0$
$y>0$

Find two positive numbers with the sum of 16 and smallest value of sum of their squares.

→ Min.

$x + y = 16$ $\qquad$ $x^2 + y^2$

$x^2 + (16-x)^2$

Let $f(x) = x^2 + (16-x)^2$ $\qquad$ $y = 16 - 8$

$f'(x) = 2x + 2(16-x)^{1} \cdot (-1)$ $\qquad$ $y = 8$

$f'(x) = 2x - 2(16-x) = 4x - 32$ → $4x - 32 = 0$

$x = 8$

$y = 8$

$f''(x) = 4 > 0$ $\qquad$ CU

Min. $(8, 128)$

↑

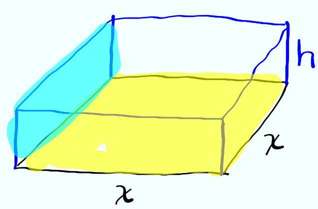
Actual value

$8^2 + (16-8)^2$

$= 64 + 64 = 128$

Oct 31-8:03 AM

A box has a square base & open top.




Volume is $32000\ \text{cm}^3$

$x \cdot x \cdot h = 32000$

$\boxed{x^2 h = 32000}$

Find dimensions with least amount of materials to make the box.

$x^2 + 4xh$ Minimize

$x^2 + 4x \cdot \dfrac{32000}{x^2}$

$f'(x)$ & $f''(x)$

$f(x) = x^2 + \dfrac{128000}{x}$

Determine Concavity & Critical point

Max

CD $\quad$ CU

Min.

Oct 31-8:10 AM

Did You look up Rolle's Theorem?

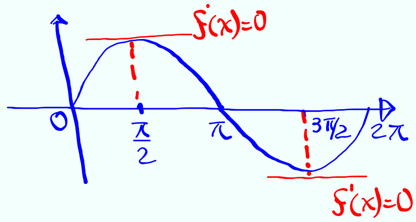
If $f(x)$ is cont. on $[a,b]$ and diff. on $(a,b)$, and $f(a)=f(b)$, then there is at least a number $c$ in $(a,b)$ such that $f'(c)=0$.

$f(x)=\sin x \quad [0,2\pi]$

$f(x)$ is cont. $[0,2\pi]$

$f'(x)=\cos x$

$f(x)$ is diff. $(0,2\pi)$




$\cos x=0 \qquad x=\frac{\pi}{2}, \; x=\frac{3\pi}{2}$

Oct 31-8:18 AM

$f(x)=3x^2-12x+5$ on $[1,3]$

$f(x)$ is polynomial $\rightarrow$ Cont. & diff. everywhere

$f(1)=3(1)^2-12(1)+5=-4$

$\rightarrow f(1)=f(3)$

$f(3)=3(3)^2-12(3)+5=-4$

Rolle's Thrm $\rightarrow f'(c)=0$ on $(1,3)$

$f'(x)=6x-12 \qquad 6c-12=0 \quad \boxed{c=2}$

Oct 31-8:24 AM

$f(x) = \sqrt{x} - \frac{1}{3}x \qquad [0,9]$

$f(x)$ is Cont. on $[0,9]$? Yes

$f'(x) = \frac{1}{2\sqrt{x}} - \frac{1}{3}$ on $(0,9)$? $f(x)$ is diff. on $(0,9)$

$f'(c) = 0 \qquad \frac{1}{2\sqrt{c}} - \frac{1}{3} = 0$

$\frac{1}{2\sqrt{c}} = \frac{1}{3} \rightarrow 2\sqrt{c} = 3$

$4c = 9$

$c = \frac{9}{4}$

Look up

Mean - Value theorem for differentiation $(0,9)$

Oct 31-8:27 AM

Looking Ahead

$f'''(x) = \cos x \rightarrow f''(x) = \sin x + C$

$f(0) = 1$

$f''(0) = \sin 0 + C = 3$

$\boxed{C = 3}$

✓$f'(0) = 2$

✓$f''(0) = 3$

find $f(x)$.

$f''(x) = \sin x + 3$

$f'(x) = -\cos x + 3x + C$

$f'(0) = -\cos 0 + 3(0) + C = 2$

$\boxed{C = 3}$

$f'(x) = -\cos x + 3x + 3$

$f(x) = -\sin x + 3 \cdot \frac{1}{2}x^2 + 3x + C$

$f(0) = -\sin 0 + \frac{3}{2}(0)^2 + 3(0) + C$

$= 1$

$\boxed{C = 1}$

$\boxed{f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1}$

Oct 31-8:32 AM